Muons and emissivities of neutrinos in neutron star cores

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Abstract

In this work we consider the role of muons in various URCA processes relevant for neutrino emissions in the core region of neutron stars. The calculations are done for β–stable nuclear matter with and without muons. We find muons to appear at densities $\rho = 0.15$ fm^{-3}, slightly around the saturation density for nuclear matter $\rho_0 = 0.16$ fm^{-3}. The direct URCA processes for nucleons are forbidden for densities below $\rho = 0.5$ fm^{-3}, however the modified URCA processes with muons $(n + N \rightarrow p + N + \mu + \overline{\nu}_\mu, p + N + \mu \rightarrow n + N + \nu_\mu)$, where N is a nucleon, result in neutrino emissivities comparable to those from $(n + N \rightarrow p + N + e + \overline{\nu}_e, p + N + e \rightarrow n + N + \nu_e)$. This opens up for further possibilities to explain the rapid cooling of neutrons stars. Superconducting protons reduce however these emissivities at densities below 0.4 fm^{-3}.

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The thermal evolution of a neutron star may provide information about the interiors of the star, and in recent years much effort has been devoted in measuring neutron star temperatures, especially with the Einstein Observatory and ROSAT, see e.g. Ref. [1]. The main cooling mechanism in the early life of a neutron star is believed to go through neutrino emissions in the core of the neutron star [2,3]. The most powerful energy losses are expected to be given by the so–called direct URCA mechanism

$$n \rightarrow p + e + \overline{\nu}_e, \qquad p + e \rightarrow n + \nu_e, \tag{1}$$

as discussed recently by several authors [2,4,5]. However, in the outer cores of massive neutron stars and in the cores of not too massive neutron stars ($M < 1.3 - 1.4 M_\odot$), the direct URCA process is allowed at densities where the momentum conservation $k_F^n < k_F^p + k_F^e$ is fulfilled[1]. This happens only at densities ρ several times the nuclear matter saturation density $\rho_0 = 0.16$ fm^{-3}.

Thus, for long time the dominant processes for neutrino emission have been the so–called modified URCA processes first discussed by Chiu and Salpeter [6], in which the two reactions

$$n + n \rightarrow p + n + e + \overline{\nu}_e, \quad p + n + e \rightarrow n + n + \nu_e, \tag{2}$$

occur in equal numbers. These reactions are just the usual processes of neutron β–decay and electron capture on protons of Eq. (1), with the addition of an extra bystander neutron. They produce neutrino–antineutrino pairs, but leave the composition of matter constant on average. Eq. (2) is referred to as the neutron branch of the modified URCA process. Another branch is the proton branch

$$n + p \rightarrow p + p + e + \overline{\nu}_e, \quad p + p + e \rightarrow n + p + \nu_e, \tag{3}$$

pointed out by Itoh and Tsuneto [7] and recently reanalyzed by Yakovlev and Levenfish [8].

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[1]Throughout this work we will reserve letters n, p, e and μ to neutrons, protons, electrons and muons respectively. Also, we will set $\hbar = c = 1$.

The latter authors showed that this process is as efficient as Eq. (2). Similarly, at higher densities, if muons are present we have the processes

$$n \rightarrow p + \mu + \overline{\nu}_\mu, \qquad p + \mu \rightarrow n + \nu_\mu, \tag{4}$$

$$n + n \rightarrow n + p + \mu + \overline{\nu}_\mu, \quad n + p + \mu \rightarrow n + n + \nu_\mu, \tag{5}$$

and

$$n + p \rightarrow p + p + \mu + \overline{\nu}_\mu, \quad p + p + \mu \rightarrow n + p + \nu_\mu, \tag{6}$$

in addition one also has the possibility of neutrino–pair bremsstrahlung, processes with baryons more massive than the nucleon participating, such as isobars or hyperons [4,5], or neutrino emission from more exotic states like pion and kaon condensates [9,10] or quark matter [11–13]. Actually, Prakash *et al.* [4] showed that the hyperon direct URCA processes gave a considerable contribution to the emissivity, without invoking exotic states or the large proton fractions needed in Eq. (1).

The scope of this Letter is to give an estimate of the processes of Eqs. (1)–(6) at densities corresponding to the outer core of massive neutron stars or the core of not too massive neutron stars, performing a self–consistent Brueckner–Hartree–Fock (BHF) calculation for β–stable nuclear matter. We impose the relevant equilibrium conditions, with and without muons, employing modern meson–exchange potential models for the nucleon–nucleon interaction. In addition, we consider also the role of superconducting protons in the core of the star, by calculating the pairing gap for protons in the $^1S_0$ state. The proton superconductivity reduces considerably the energy losses in the above reactions [8], and may have important consequences for the cooling of young neutron stars. The final outcome of our calculations yields then the equation of state, effective masses m^* for protons and nucleons, their corresponding Fermi momenta k_F, the Fermi momenta for electrons and muons and the proton pairing gap. These ingredients enter into the calculation of the various reaction rates.

In more detail, our calculational procedure is as follows:

First we solve self–consistently the BHF equations for the single-particle energies, using a G–matrix defined through the Bethe–Brueckner–Goldstone equation as

$$G = V + V \frac{Q}{\omega - H_0} G, \tag{7}$$

where V is the nucleon-nucleon potential, Q is the Pauli operator which prevents scattering into intermediate states prohibited by the Pauli principle, H_0 is the unperturbed hamiltonian acting on the intermediate states and ω is the so–called starting energy, the unperturbed energy of the interacting states. Methods to solve this equation are reviewed in Ref. [14]. The single–particle energies for state k_i (i encompasses all relevant quantum numbers like momentum, isospin projection, spin etc.) in nuclear matter are assumed to have the simple quadratic form

$$\varepsilon_{k_i} = \frac{k_i^2}{2m^*} + \delta_i, \tag{8}$$

where m^* is the effective mass. The terms m^* and δ, the latter being an effective single–particle potential related to the G–matrix, are obtained through the self–consistent BHF procedure. The so–called model–space BHF method for the single–particle spectrum has been used, see e.g. Refs. [14], with a cutoff $k_M = 3.0$ fm^{-1}. This self–consistency scheme consists in choosing adequate initial values of the effective mass and δ. The obtained G–matrix is in turn used to obtain new values for m^* and δ for protons and neutrons. This procedure continues until these parameters vary little. The BHF equations are solved for different proton fractions, using the formalism of Refs. [14,15]. The conditions of β equilibrium require that

$$\mu_n = \mu_p + \mu_e, \tag{9}$$

where μ_i is the chemical potential and that charge is conserved

$$\rho_p = \rho_e, \tag{10}$$

where ρ_i is the particle number density in fm^{-3} for particle i. If muons are present, we need also to satisfy charge conservation

$$\rho_p = \rho_e + \rho_\mu, \tag{11}$$

and energy conservation through

$$\mu_e = \mu_\mu. \tag{12}$$

The nucleon–nucleon potential is defined by the parameters of the meson–exchange potential model of the Bonn group, version A in Table A.2 of Ref. [16]. From the derived effective interaction one can in turn calculate the energy per nucleon as function of the total density [14]. This is plotted in Fig. 1 with and without muons. As can be seen from this figure, muons yield contributions to the energy per particle (and in Fig. 2 to the pairing gap) at densities around 0.15 fm^{-3}. The presence of muons makes the energy per particle and in turn the equation of state softer, yielding smaller masses and larger radii ($M = 1.45 M_\odot$ and $R \approx 9$ km here) for neutron stars. Similar qualitative results are reached if one performs a Dirac–Brueckner–Hartree-Fock calculation, as in Ref. [17], although the equation of state in that case is stiffer, resulting in larger masses and smaller radii for the star. More details and applications to neutron stars will be reported by us in a future work [18].

The next step is to evaluate the gap equation following the scheme proposed by Anderson and Morel [19] and applied to nuclear physics by Baldo et $al.$ [20]. These authors introduced an effective interaction $\tilde{V}_{k,k'}$. This effective interaction sums up all two–particle excitations above the cutoff k_M. It is defined according to

$$\tilde{V}_{k,k'} = V_{k,k'} - \sum_{k''>k_M} V_{k,k''} \frac{1}{2E_{k''}} \tilde{V}_{k'',k'}, \tag{13}$$

where the quasiparticle energy E_k is given by

$$E_k = \sqrt{(\varepsilon_k - \varepsilon_F)^2 + \Delta_k^2}, \tag{14}$$

ε_F being the single–particle energy at the Fermi surface, $V_{k,k'}$ is the free nucleon–nucleon potential in momentum space and Δ_k is the pairing gap

$$\Delta_k = -\sum_{k' \le k_M} \tilde{V}_{k,k'} \frac{\Delta_{k'}}{2E_{k'}}. \tag{15}$$

For notational economy, we have dropped the subscript i on the single–particle energies.

In summary, first we obtain the self–consistent BHF single–particle spectrum ε_k for protons and neutrons in β–stable matter. This procedure gives the relevant proton and neutron fractions at a given total density. Thereafter we solve self–consistently Eqs. (13) and (15) in order to obtain the pairing gap Δ for protons or neutrons. For states above k_M, the quasiparticle energy of (14) is approximated by $E_k = (\varepsilon_k - \varepsilon_F)$, an approximation found to yield satisfactory results in neutron matter [21]. Our prescription gives then the pairing gap at $T = 0$. The results for the proton pairing gap with and without muons are displayed in Fig. 2 as functions of the total baryonic density.

The reader should notice that our calculation of the pairing gap does not include complicated many–body contributions like polarization terms. These may further reduce the pairing gap and bring in an uncertainty regarding the size, which strongly influences the modified URCA processes. Although we have employed the Bonn A potential in Table A.2 of Ref. [16], the pairing gap obtained with other potentials like the Paris or Bonn B and Bonn C is rather similar. results. This is expected since the $^1S_0$ channel is determined by the central force component only of the nucleon–nucleon interaction, and all modern potentials reproduce the phase shifts for this channel.

With the pairing gap, effective masses and the Fermi momenta for the various particles, we first look at the neutrino energy production rate for Eq. (2). The expressions for these processes were derived by Friman and Maxwell [22] and read [8]

$$Q_n \approx 8.5 \times 10^{21} \left(\frac{m_n^*}{m_n}\right)^3 \left(\frac{m_p^*}{m_p}\right) \left(\frac{\rho_e}{\rho_0}\right)^{1/3} T_9^8 \alpha_n \beta_n, \tag{16}$$

in units of ergs cm^{-3}s^{-1} where T_9 is the temperature in units of 10^9 K, and according to Friman and Maxwell α_n describes the momentum transfer dependence of the squared matrix element in the Born approximation for the production rate in the neutron branch. Similarly, β_n includes the non–Born corrections and corrections due to the nucleon–nucleon interaction

not described by one–pion exchange. Friman and Maxwell [22] used $\alpha_n \approx 1.13$ at nuclear matter saturation density and $\beta_n = 0.68$. In the results presented below, we will not include α_n and β_n. For the reaction of Eq. (5), one has to replace ρ_e with ρ_μ. For the proton branch of Eq. (3) we have the approximate equation [8]

$$Q_p \approx 8.5 \times 10^{21} \left(\frac{m_p^*}{m_p}\right)^3 \left(\frac{m_n^*}{m_n}\right) \left(\frac{\rho_e}{\rho_0}\right)^{1/3}$$
$$\times T_9^8 \alpha_p \beta_p \left(1 - \frac{k_F^e}{4k_F^p}\right) \Theta, \tag{17}$$

where $\Theta = 1$ if $k_F^n < 3k_F^p + k_F^e$ and zero elsewhere. Yakovlev and Levenfish put $\alpha_n = \alpha_p$ and $\beta_p = \beta_n$. We will, due to the uncertainty in the determination of these coefficients, omit them in our calculations of the reaction rates. For Eq. (6) one replaces ρ_e with ρ_μ and k_F^e with k_F^μ.

The reaction rates for the URCA processes are reduced due to the superconducting protons. Here we adopt the results from Yakovlev and Levenfish [8], their Eqs. (31) and (32) for the neutron branch of Eq. (16) and Eqs. (35) and (37) for the proton branch of Eq. (17). In this work we study proton singlet–superconductivity only, employing the standard approximation [23]

$$\frac{k_B T_C}{\Delta(0)} = 0.5669, \tag{18}$$

where T_C is the critical temperature and $\Delta(0)$ is the pairing gap at zero temperature derived in our calculations. The critical temperature is then used to obtain the temperature dependence of the corrections to the neutrino reaction rates due to superconducting protons. To achieve that we employ Eq. (23) of [8].

In this Letter, we present results for the neutrino energy rates at a density $\rho = 0.3$ fm^{-3}. The critical temperature is $T_C = 3.993 \times 10^9$ K, whereas without muons we have $T_C = 4.375 \times 10^9$. The implications for the final neutrino rates are shown in Fig. 3, where we show the results for the full case with both muons and electrons for the processes of Eqs. (2), (3), (5) and (6). In addition, we also display the results when there is no reduction due to superconducting protons. At the density considered, $\rho = 0.3$ fm^{-3}, we see that the

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processes of Eqs. (5) and (6) are comparable in size to those of Eqs. (2) and (3) (on the log–scale there is basically no difference). The direct URCA process of Eq. (1) is not allowed in our β–stable matter for densities $\rho < 0.5$ fm^{-3}, due to momentum conservation. Similarly, the direct URCA process with muons is also not allowed at densities $\rho < 0.6$ fm^{-3}. Thus, Eqs. (5) and (6) give an additional and important contribution to the neutrino production in the core of a neutron star. However, the proton pairing gap is still sizable at densities up to 0.4 fm^{-3}, and yields a significant suppression of the modified URCA processes discussed here, as seen in Fig. 3. We have omitted any discussion on neutron pairing in the $^3P_2$ state. For this channel we find the pairing gap to be rather small [21], less than 0.1 MeV and close to that obtained in Ref. [24]. We expect therefore that the major reductions of the neutrino rates in the core come from superconducting protons in the $^1S_0$ state. Moreover, the recent reinvestigation of neutrino–pair bremsstrahlung in the crust by Pethick and Thorsson [25] indicates that this process is much less important for the thermal evolution of neutrons stars than suggested by earlier calculations. Thus, combined with the present reduction due to superconducting protons, our results may indicate that there is need for other processes than those studied here to explain the rapid cooling of neutron stars. The contribution from neutrino–pair bremsstrahlung in nucleon–nucleon collisions in the core is also, for most temperature ranges relevant for neutron stars, smaller than the contribution from modified URCA processes [8]. Possible candidates are then direct URCA processes due to hyperons and isobars, as suggested in Ref. [4], or neutrino production through exotic states of matter, like kaon or pion condensation [9,10] or quark matter [11–13]. Though, the analysis of Page [3] indicates that it is hard to discriminate between fast or slow cooling scenarios, though in both cases agreement with the observed temperature of Geminga is obtained if baryon pairing is present in most, if not all of the core of the star.

In summary, we have performed a self–consistent calculation for β–stable neutron matter, with and without muons. We have shown that the modified URCA processes ($n + N \rightarrow p + N + \mu + \overline{\nu}_\mu, p + N + \mu \rightarrow n + N + \nu_\mu$), where N is a nucleon, result in neutrino emissivities comparable to those from ($n + N \rightarrow p + N + e + \overline{\nu}_e, p + N + e \rightarrow n + N + \nu_e$). These processes

should therefore be accounted for in a scenario for neutron star cooling.

REFERENCES

[1] H. Ögelman, in Proceedings of the Nato ASI on "The Lives of the Neutron Stars", eds. A. Alpar, U. Kiziloglu and J. van Paradijs, (Kluwer, Amsterdam, 1995), 101.

[2] C. J. Pethick, Rev. Mod. Phys. **64**, 1133 (1992).

[3] D. Page, ApJ **428**, 250 (1994).

[4] M. Prakash, M. Prakash, J. M. Lattimer and C. J. Pethick, ApJ **390**, L77 (1992).

[5] M. Prakash, Phys. Reports **242**, 191 (1994).

[6] H.-J. Chiu and E. E. Salpeter, Phys. Rev. Lett. **12**, 413 (1964).

[7] I. Itoh and T. Tsuneto, Prog. Theor. Phys. **48**, 149 (1972)

[8] D. G. Yakovlev and K. P. Levenfish, Astron. Astrophys. **297**, 717 (1995).

[9] G. E. Brown, C.–H. Lee, M. Rho and V. Thorsson, Nucl. Phys. **A567**, 937 (1994); V. Thorsson, M. Prakash and J. M. Lattimer, Nucl. Phys. **A572**, 693 (1994).

[10] A. B. Migdal, E. E. Saperstein, M. A. Troitsky and D. N. Voskresensky, Phys. Rep. **192**, 179 (1990).

[11] N. K. Glendenning, in Proceedings of the International Summer School on "Structure of Hadrons and Hadronic Matter", (World Scientific, Singapore, 1991), 275.

[12] N. Iwamoto, Ann. Phys. **141**, 1 (1982).

[13] A. Drago, U. Tambini and M. Hjorth–Jensen, ECT* preprint ECT*/MAY/95–03.

[14] M. Hjorth–Jensen, T. T. S. Kuo and E. Osnes, Phys. Reports **261**, 125 (1995).

[15] H. Q. Song, Z. X. Wang and T. T. S. Kuo, Phys. Rev. C **46**, 1788 (1992).

[16] R. Machleidt, Adv. Nucl. Phys. **19**, 185 (1989).

[17] L. Engvik, M. Hjorth–Jensen, E. Osnes, G. Bao and E. Østgaard, Phys. Rev. Lett. **73**,

2650 (1994).

[18] Ø. Elgarøy, L. Engvik, E. Osnes, F. V. De Blasio, M. Hjorth–Jensen and G. Lazzari, unpublished.

[19] P. W. Anderson and P. Morel, Phys. Rev. **123**, 1911 (1961).

[20] M. Baldo, J. Cugnon, A. Lejeune and U. Lombardo, Nucl. Phys. **A515**, 409 (1990).

[21] Ø. Elgarøy, MSc. Thesis, University of Oslo (1995), unpublished.

[22] B. L. Friman and O. V. Maxwell, Ap. J. **232**, 541 (1979).

[23] see e.g. L. Amundsen and E. Østgaard, Nucl. Phys. **A437**, 487 (1985); Nucl. Phys. **A442**, 163 (1985) .

[24] T. Takatsuka and R. Tamagaki, Prog. Theor. Phys. Suppl. **112**, 27 (1993).

[25] C. J. Pethick and V. Thorsson, Phys. Rev. Lett. **72**, 1964 (1994).

FIG. 1. Energy per baryon as function of the total baryonic density ρ in β–stable neutron matter with (dashed line) and without (solid line) muons.

FIG. 2. Proton pairing energy gap as function of total baryonic density ρ in β–stable neutron matter with (dashed line) and without (solid line) muons.

FIG. 3. Temperature dependence of neutrino energy loss rates in a neutron star core at a total baryonic density of 0.3 fm^{-3} with muons and electrons. Solid line represents Eq. (2), dotted line is Eq. (5), dashed line is Eq. (3) while the dash–dotted line is the processes of Eq. (6). The corresponding results with no pairing are also shown.
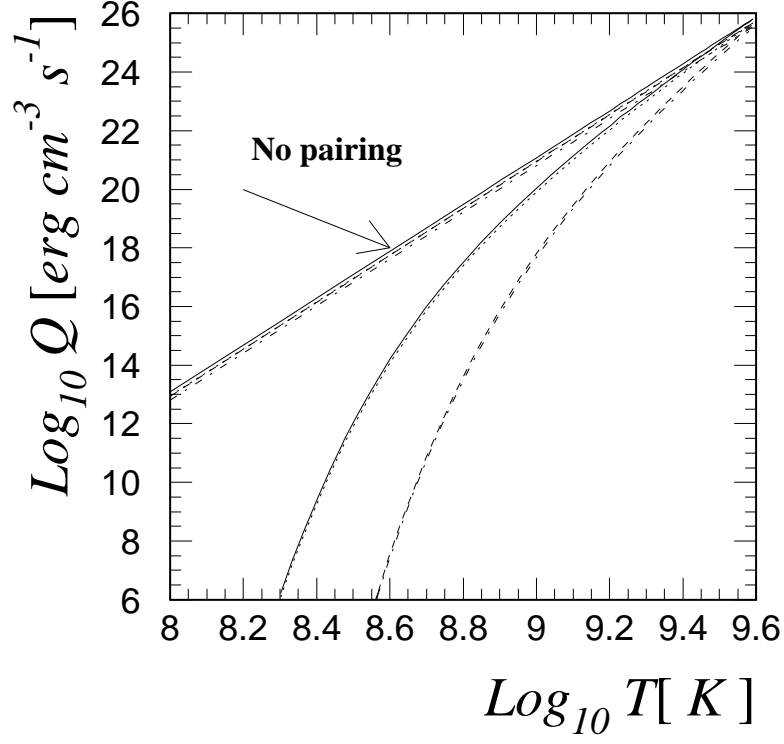
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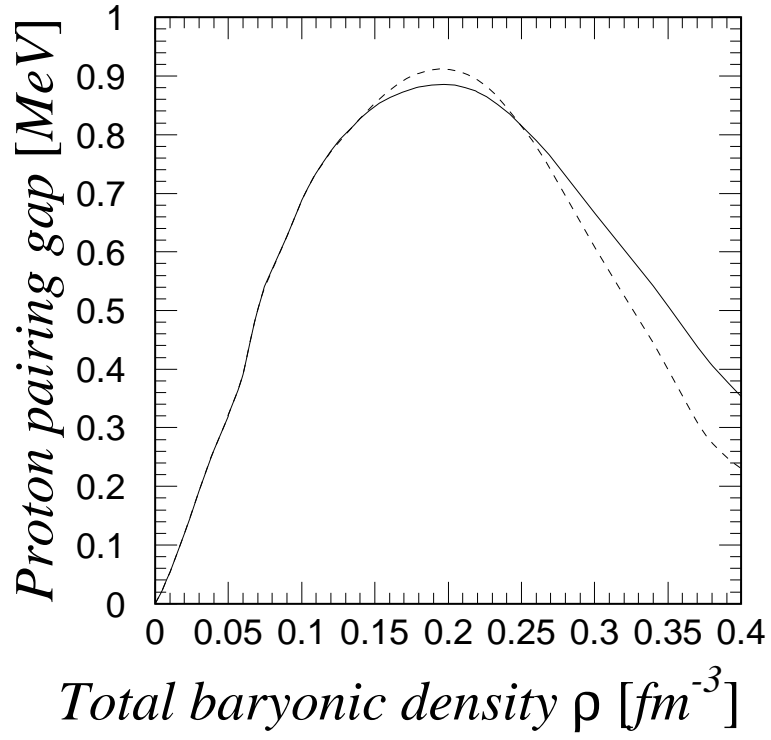
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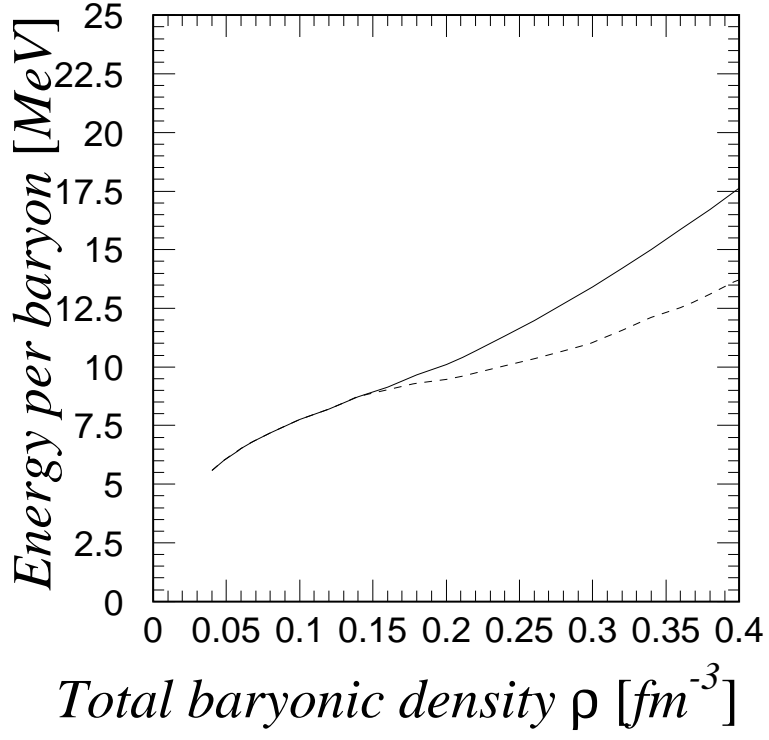
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